Normal Distribution [302 marks]

1. [Maximum mark: 15]

The length, X mm, of a certain species of seashell is normally distributed with mean 25 and variance, σ^2.

The probability that X is less than 24.15 is 0.1446.

(a) Find $\mathrm{P}(24.15 < X < 25)$. [2]

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(b.i) Find σ, the standard deviation of X. [3]

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(b.ii) Hence, find the probability that a seashell selected at random has a length greater than 26 mm. [2]

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A random sample of 10 seashells is collected on a beach. Let Y represent the number of seashells with lengths greater than 26 mm.

(c) Find $E(Y)$. [3]

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(d) Find the probability that exactly three of these seashells have a length greater than 26 mm. [2]

(e) A seashell selected at random has a length less than 26 mm.

Find the probability that its length is between 24.15 mm and 25 mm. [3]

2. [Maximum mark: 15]

The time, T minutes, taken to complete a jigsaw puzzle can be modelled by a normal distribution with mean μ and standard deviation 8.6.

It is found that 30% of times taken to complete the jigsaw puzzle are longer than 36.8 minutes.

(a) By stating and solving an appropriate equation, show, correct to two decimal places, that $\mu = 32.29$. [4]

Use $\mu = 32.29$ in the remainder of the question.

(b) Find the 86th percentile time to complete the jigsaw puzzle. [2]

(c) Find the probability that a randomly chosen person will take more than 30 minutes to complete the jigsaw puzzle. [2]

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Six randomly chosen people complete the jigsaw puzzle.

(d) Find the probability that at least five of them will take more than 30 minutes to complete the jigsaw puzzle. [3]

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(e) Having spent 25 minutes attempting the jigsaw puzzle, a randomly chosen person had not yet completed the puzzle.

Find the probability that this person will take more than 30 minutes to complete the jigsaw puzzle. [4]

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3. [Maximum mark: 4]

The random variable Z has a standard normal distribution with mean 0 and standard deviation 1.

(a) Find $\mathrm{P}(Z < 1.7)$. [1]

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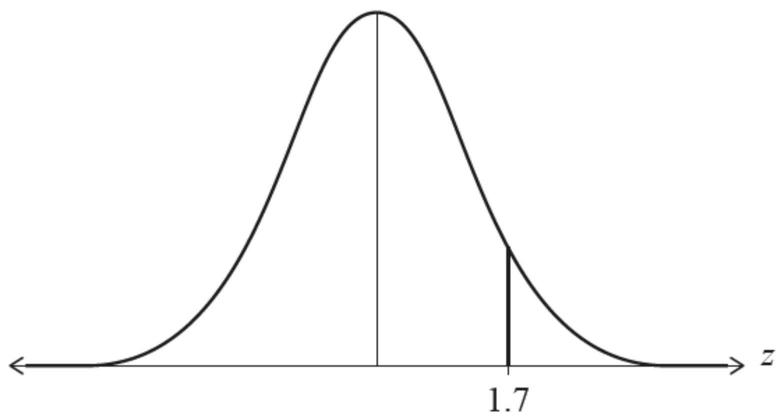
It is known that $\mathrm{P}(a < Z < 1.7) = 0.7865$, correct to four significant figures.

(b) Sketch this information on the following diagram.



[1]

(c) Hence, or otherwise, find the value of a. [2]

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4. [Maximum mark: 7]

A garden centre sells seeds for two varieties of sunflowers: large and giant.

The heights of large sunflowers are normally distributed with mean μ and standard deviation σ.

It is known that 25% of large sunflowers have a height of over $180\ \mathrm{cm}$.

(a) Given that $\mu + A\sigma = 180$, where $A \in \mathbb{R}$, find the value of A. [3]

The heights of giant sunflowers are also normally distributed.

The giant sunflowers have a mean height that is $35\ \mathrm{cm}$ more than that of the large sunflowers and a standard deviation that is double that of the large sunflowers.

It is known that 98% of giant sunflowers have a height greater than $180\ \mathrm{cm}$.

(b) Determine μ and σ. [4]

5. [Maximum mark: 6]

The random variables X and Y are normally distributed with $X \sim \mathrm{N}\left(7,\ a^2\right)$ and $Y \sim \mathrm{N}\left(19,\ a^2\right)$, where $a > 0$.

(a) Find b such that $\mathrm{P}(X > b) = \mathrm{P}(Y > 22)$. [2]

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(b) Write down the approximate value of $\mathrm{P}(7 - a < X < 7 + a)$, correct to two significant figures. [1]

(c) Given that $a = 3$, calculate the approximate value of $\mathrm{P}(Y < 22)$, correct to two significant figures. [3]

6. [Maximum mark: 8]

In a study, measurements for arm span, A cm, and foot length, F cm, are taken from a large group of adults.

For this group, the regression line of F on A is found to be $F = 0.335A - 32.6$, and the regression line of A on F is found to be $A = 2.89F + 99.3$. Each regression line passes through the mean point.

(a) By using an appropriate regression line, find an estimate of the arm span for an adult with a foot length of 19.8 cm. [2]

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(b) For this group of adults, find the mean arm span and the mean foot length. [3]

The heights, H cm, of adults in the group can be modelled by a normal distribution with mean 163 cm and standard deviation σ cm.

It is found that 88% of the group have a height between 153 cm and 173 cm.

(c) Find the value of σ. [3]

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7. [Maximum mark: 17]

At Adam's Apple Orchard the weights of apples, W, in grams, are normally distributed with a mean 175 grams and standard deviation 8 grams.

(a) Find the probability that a randomly chosen apple weighs less than 170 grams. [2]

(b) It is found that $20\,\%$ of the apples weigh more than w grams. Find w, correct to four significant figures. [2]

All orchards classify an apple as premium when its weight is between 170 and 185 grams.

(c) Find the percentage of apples that are classified as premium at Adam's Apple Orchard. [2]

After orders are completed, there are many apples left over. Boxes are filled with randomly chosen left-over apples. Each box contains 40 apples.

(d) Find the probability that a randomly chosen box contains at least 30 premium apples. [3]

(e) If 10 of these boxes are randomly selected, find the probability that exactly 4 boxes have at least 30 premium apples. [2]

At a neighbouring orchard the weights of apples, M, in grams, are normally distributed with mean μ and standard deviation σ. It is known that:

- $82\,\%$ of their apples are classified as premium
- the percentage of apples that weigh less than 170 grams is twice the percentage of apples that weigh more than 185 grams.

(f) Find the value of μ. [6]

8. [Maximum mark: 5]

A farmer grows two types of apples—cooking apples and eating apples. The weights of the apples, in grams, can be modelled as normal distributions with the following parameters.

Apple type	Mean μ	Standard deviation σ
Eating	$100\,\text{g}$	$20\,\text{g}$
Cooking	$140\,\text{g}$	$40\,\text{g}$

For each type of apple you can assume that $95\,\%$ of the weights are within two standard deviations of the mean.

(a) Find the percentage of eating apples that have a weight greater than $140\,\text{g}$. [1]

The farmer grows a large number of apples of which 80% are eating apples.

Both types of apples are picked and randomly mixed together in a cleaning machine.

After cleaning, the machine separates out those that have a weight greater than $140\,\mathrm{g}$ into a container.

(b) An apple is randomly selected from this container. Find the probability that it is an eating apple. Give your answer in the form $\frac{c}{d}$, where $c,\ d \in \mathbb{Z}^+$. [4]

9. [Maximum mark: 14]

A lake contains a type of fish called carp. The lengths, L cm, of the carp can be modelled by a normal distribution with mean 45.6 cm and standard deviation 4.2 cm.

According to this model, carp with a length between 41.4 cm and k cm lie within one standard deviation of the mean.

(a) Write down the value of k. [2]

(b) Find the probability that a randomly selected carp is greater than 48 cm in length. [2]

(c) It is known that 99% of carp in the lake have a length greater than x cm. Find the value of x. [2]

(d) Consider a random sample of 100 carp from the lake.

(d.i) Find the expected number of carp with lengths between 40 cm and 56 cm. [3]

(d.ii) Find the probability that in this sample, exactly 95 carp have a length between 40 cm and 56 cm. [2]

A large sample of carp from the lake is studied. The length of each fish is measured and recorded correct to the nearest 0.1 cm.

(e) Find the probability that a randomly selected carp has a length recorded as 45.6 cm. [3]

10. [Maximum mark: 4]

The random variable X is normally distributed with mean 10 and standard deviation 2.

(a) Find the probability that X is more than 1.5 standard deviations above the mean. [2]

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The probability that X is more than k standard deviations above the mean is 0.1, where $k \in \mathbb{R}$.

(b) Find the value of k. [2]

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11. [Maximum mark: 31]

This question asks you to find the probability of graphs of randomly generated quadratic functions having a specified number of x-intercepts.

In parts (a) – (f), consider quadratic functions, $f(x) = ax^2 + bx + c$, whose coefficients, $\mathrm{a},\ \mathrm{b}$ and c, are randomly generated in turn by rolling an unbiased six-sided die three times and reading off the value shown on the uppermost face of the die.

For example, rolling a $2,\ 3$ and 5 in turn generates the quadratic function $f(x) = 2x^2 + 3x + 5$.

(a) Explain why there are 216 possible quadratic functions that can be generated using this method. [1]

(b) The set of coefficients, $a = 1,\ b = 4$ and $c = 4$, is randomly generated to form the quadratic function $f(x) = x^2 + 4x + 4$

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[2]

Verify that this graph of f has only one x-intercept.

(c) By considering the discriminant, or otherwise, show that the probability of the graph of such a randomly generated quadratic function having only one x-intercept is $\frac{5}{216}$. [6]

Now consider randomly generated quadratic functions whose corresponding graphs have two **distinct** x-intercepts.

(d) By considering the discriminant, determine the set of possible values of ac. [3]

(e.i) For the case where $ac = 1$, show that there are four quadratic functions whose corresponding graphs have two distinct x-intercepts. [1]

(e.ii) For the case where $ac = 2$, show that there are eight quadratic functions whose corresponding graphs have two distinct x-intercepts. [2]

Let p be the probability of the graph of such a randomly generated quadratic function having two distinct x-intercepts.

(f) Using the approach started in part (e), or otherwise, find the value of p. [6]

In parts (g) and (h), consider a randomly generated quadratic function, $f(x) = x^2 + 2Zx + 1$, where the continuous random variable $Z \sim \mathrm{N}(0,\ 1)$.

(g) Find the probability that the graph of f has two x-intercepts. [3]

The continuous random variables, X_1 and X_2, represent the x-intercepts of the graph of f where $X_1 = -Z - \sqrt{Z^2 - 1}$ and $X_2 = -Z + \sqrt{Z^2 - 1}$.

(h) Given that the graph of f has two x-intercepts, X_1 and X_2, find the probability that both X_1 and X_2 are greater than 0.5. [7]

12. [Maximum mark: 16]

A farmer is growing a field of rice plants. The height, H cm, of each plant can be modelled by a normal distribution with mean μ and standard deviation σ.

It is known that $\mathrm{P}(H < 82.4) = 0.213$ and $\mathrm{P}(H > 87.3) = 0.409$.

(a) Find the probability that the height of a randomly selected plant is between 82.4 cm and 87.3 cm. [2]

(b) Find the value of μ and the value of o. [5]

The farmer measures 100 randomly selected plants. Any plant with a height greater than 87.3 cm is considered ready to harvest. Heights of plants are independent of each other.

(c.i) Find the probability that exactly 32 plants are ready to harvest. [2]

(c.ii) Given that fewer than 44 plants are ready to harvest, find the probability that exactly 32 plants are ready to harvest. [4]

In another field, the farmer is growing the same variety of rice, but is using a different fertilizer. The heights of these plants, F cm, are normally distributed with mean 92.8 and standard deviation d. The farmer finds the interquartile range to be 4.52 cm.

(d) Find the value of d. [3]

13. [Maximum mark: 16]

A farmer is growing a field of wheat plants. The height, H cm, of each plant can be modelled by a normal distribution with mean μ and standard deviation σ.

It is known that $\mathrm{P}(H < 94.6) = 0.288$ and $\mathrm{P}(H > 98.1) = 0.434$.

(a) Find the probability that the height of a randomly selected plant is between 94.6 cm and 98.1 cm. [2]

(b) Find the value of μ and the value of σ. [5]

The farmer measures 100 randomly selected plants. Any plant with a height greater than 98.1 cm is considered ready to harvest. Heights of plants are independent of each other.

(c.i) Find the probability that exactly 34 plants are ready to harvest. [2]

(c.ii) Given that fewer than 49 plants are ready to harvest, find the probability that exactly 34 plants are ready to harvest. [4]

In another field, the farmer is growing the same variety of wheat, but is using a different fertilizer. The heights of these plants, F cm, are normally distributed with mean 98.6 and standard deviation d. The farmer finds the interquartile range to be 4.82 cm.

(d) Find the value of d. [3]

14. [Maximum mark: 5]

A company manufactures metal tubes for bicycle frames. The diameters of the tubes, D mm, are normally distributed with mean 32 and standard deviation σ. The interquartile range of the diameters is 0.28.

Find the value of σ. [5]

15. [Maximum mark: 16]

The time worked, T, in hours per week by employees of a large company is normally distributed with a mean of 42 and standard deviation 10.7.

(a) Find the probability that an employee selected at random works more than 40 hours per week. [2]

(b) A group of four employees is selected at random. Each employee is asked in turn whether they work more than 40 hours per week. Find the probability that the fourth employee is the only one in the group who works more than 40 hours per week. [3]

A large group of employees work more than 40 hours per week.

(c.i) An employee is selected at random from this large group.

Find the probability that this employee works less than 55 hours per week. [4]

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(c.ii) Ten employees are selected at random from this large group.

Find the probability that exactly five of them work less than 55 hours per week. [3]

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It is known that $\mathrm{P}(a \leq T \leq b) = 0.904$ and that $\mathrm{P}(T > b) = 2\mathrm{P}(T < a)$, where a and b are numbers of hours worked per week. An employee who works fewer than a hours per week is considered to be a part-time employee.

(d) Find the maximum time, in hours per week, that an employee can work and still be considered part-time. [4]

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16. [Maximum mark: 16]

A bakery makes two types of muffins: chocolate muffins and banana muffins.

The weights, C grams, of the chocolate muffins are normally distributed with a mean of $62\,\text{g}$ and standard deviation of $2.9\,\text{g}$.

(a) Find the probability that a randomly selected chocolate muffin weighs less than $61\,\text{g}$. [2]

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(b) In a random selection of 12 chocolate muffins, find the probability that exactly 5 weigh less than $61\,\text{g}$. [2]

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The weights, B grams, of the banana muffins are normally distributed with a mean of $68\,\text{g}$ and standard deviation of $3.4\,\text{g}$.

Each day 60% of the muffins made are chocolate.

On a particular day, a muffin is randomly selected from all those made at the bakery.

(c.i) Find the probability that the randomly selected muffin weighs less than $61\,\text{g}$. [4]

(c.ii) Given that a randomly selected muffin weighs less than $61\,\mathrm{g}$, find the probability that it is chocolate. [3]

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The machine that makes the chocolate muffins is adjusted so that the mean weight of the chocolate muffins remains the same but their standard deviation changes to σ g. The machine that makes the banana muffins is not adjusted. The probability that the weight of a randomly selected muffin from these machines is less than 61 g is now 0.157.

(d) Find the value of σ. [5]

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17. [Maximum mark: 18]

The time it takes Suzi to drive from home to work each morning is normally distributed with a mean of 35 minutes and a standard deviation of σ minutes.

On 25% of days, it takes Suzi longer than 40 minutes to drive to work.

(a) Find the value of σ. [4]

(b) On a randomly selected day, find the probability that Suzi's drive to work will take longer than 45 minutes. [2]

Suzi will be late to work if it takes her longer than 45 minutes to drive to work. The time it takes to drive to work each day is independent of any other day.

Suzi will work five days next week.

(c) Find the probability that she will be late to work at least one day next week. [3]

(d) Given that Suzi will be late to work at least one day next week, find the probability that she will be late less than three times. [5]

Suzi will work 22 days this month. She will receive a bonus if she is on time at least 20 of those days.

So far this month, she has worked 16 days and been on time 15 of those days.

(e) Find the probability that Suzi will receive a bonus. [4]

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18. [Maximum mark: 7]

Rachel and Sophia are competing in a javelin-throwing competition.

The distances, R metres, thrown by Rachel can be modelled by a normal distribution with mean 56.5 and standard deviation 3.

The distances, S metres, thrown by Sophia can be modelled by a normal distribution with mean 57.5 and standard deviation 1.8.

In the first round of competition, each competitor must have five throws. To qualify for the next round of competition, a competitor must record at least one throw of 60 metres or greater in the first round.

Find the probability that only one of Rachel or Sophia qualifies for the next round of competition. [7]

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19. [Maximum mark: 16]

The random variable X follows a normal distribution with mean μ and standard deviation σ.

(a) Find $\mathrm{P}(\mu - 1.5\sigma < X < \mu + 1.5\sigma)$. [3]

The avocados grown on a farm have weights, in grams, that are normally distributed with mean μ and standard deviation σ. Avocados are categorized as small, medium, large or premium, according to their weight. The following table shows the probability an avocado grown on the farm is classified as small, medium, large or premium.

Category	Small	Medium	Large	Premium
Probability	0.04	0.576	0.288	0.096

The maximum weight of a small avocado is 106.2 grams.

The minimum weight of a premium avocado is 182.6 grams.

(b) Find the value of μ and of σ. [5]

A supermarket purchases all the avocados from the farm that weigh more than 106.2 grams.

Find the probability that an avocado chosen at random from this purchase is categorized as

(c.i) medium. [2]

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(c.iii) premium. [1]

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(d) The selling prices of the different categories of avocado at this supermarket are shown in the following table:

Category	Medium	Large	Premium
Selling price ($) per avocado	1.10	1.29	1.96

The supermarket pays the farm $\$200$ for the avocados and assumes it will then sell them in exactly the same proportion as purchased from the farm.

According to this model, find the minimum number of avocados that must be sold so that the net profit for the supermarket is at least $\$438$. [4]

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20. [Maximum mark: 6]

A company produces bags of sugar whose masses, in grams, can be modelled by a normal distribution with mean 1000 and standard deviation 3.5. A bag of sugar is rejected for sale if its mass is less than 995 grams.

(a) Find the probability that a bag selected at random is rejected. [2]

(b) Estimate the number of bags which will be rejected from a random sample of 100 bags. [1]

(c) Given that a bag is not rejected, find the probability that it has a mass greater than 1005 grams. [3]

21. [Maximum mark: 15]

The flight times, T minutes, between two cities can be modelled by a normal distribution with a mean of 75 minutes and a standard deviation of σ minutes.

(a) Given that 2% of the flight times are longer than 82 minutes, find the value of σ. [3]

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(b) Find the probability that a randomly selected flight will have a flight time of more than 80 minutes. [2]

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(c) Given that a flight between the two cities takes longer than 80 minutes, find the probability that it takes less than 82 minutes. [4]

On a particular day, there are 64 flights scheduled between these two cities.

(d) Find the expected number of flights that will have a flight time of more than 80 minutes. [3]

(e) Find the probability that more than 6 of the flights on this particular day will have a flight time of more than 80 minutes. [3]

22. [Maximum mark: 17]

Emlyn plays many games of basketball for his school team. The number of minutes he plays in each game follows a normal distribution with mean m minutes.

In any game there is a $30\,\%$ chance he will play less than $13.\,6\ \mathrm{minutes}$.

(a) Sketch a diagram to represent this information. [2]

In any game there is a $70\,\%$ chance he will play less than $17.\,8\ \mathrm{minutes}$.

(b) Show that $m = 15.7$. [2]

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The standard deviation of the number of minutes Emlyn plays in any game is 4.

(c.i) Find the probability that Emlyn plays between 13 minutes and 18 minutes in a game. [2]

(c.ii) Find the probability that Emlyn plays more than 20 minutes in a game. [2]

There is a $60\,\%$ chance Emlyn plays less than x minutes in a game.

(d) Find the value of x. [2]

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Emlyn will play in two basketball games today.

(e) Find the probability he plays between 13 minutes and 18 minutes in one game and more than 20 minutes in the other game. [3]

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Emlyn and his teammate Johan each practise shooting the basketball multiple times from a point X. A record of their performance over the weekend is shown in the table below.

	Emlyn	**Johan**
Saturday	42 successful shots from 70 attempts	16 successful shots from 30 attempts
Sunday	27 successful shots from 32 attempts	51 successful shots from 68 attempts

On Monday, Emlyn and Johan will practise and each will shoot 200 times from point X.

(f) Find the expected number of successful shots Emlyn will make on Monday, based on the results from Saturday and Sunday. [2]

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(g) Emlyn claims the results from Saturday and Sunday show that his expected number of successful shots will be more than Johan's.

Determine if Emlyn's claim is correct. Justify your reasoning. [2]

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23. [Maximum mark: 14]

Fiona walks from her house to a bus stop where she gets a bus to school. Her time, W minutes, to walk to the bus stop is normally distributed with $W \sim \mathrm{N}\left(12,\ 3^2\right)$.

Fiona always leaves her house at 07:15. The first bus that she can get departs at 07:30.

(a) Find the probability that it will take Fiona between 15 minutes and 30 minutes to walk to the bus stop. [2]

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The length of time, B minutes, of the bus journey to Fiona's school is normally distributed with $B \sim \mathrm{N}\left(50,\ \sigma^2\right)$. The probability that the bus journey takes less than 60 minutes is 0.941.

(b) Find σ. [3]

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(c) Find the probability that the bus journey takes less than 45 minutes. [2]

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If Fiona misses the first bus, there is a second bus which departs at 07:45. She must arrive at school by 08:30 to be on time. Fiona will not arrive on time if she misses both buses. The variables W and B are independent.

(d) Find the probability that Fiona will arrive on time. [5]

(e) This year, Fiona will go to school on 183 days.

Calculate the number of days Fiona is expected to arrive on time. [2]

24. [Maximum mark: 14]

The weights, in grams, of individual packets of coffee can be modelled by a normal distribution, with mean $102\,\text{g}$ and standard deviation $8\,\text{g}$.

(a) Find the probability that a randomly selected packet has a weight less than $100\,\text{g}$. [2]

(b) The probability that a randomly selected packet has a weight greater than w grams is 0.444. Find the value of w. [2]

(c) A packet is randomly selected. Given that the packet has a weight greater than $105\,\text{g}$, find the probability that it has a weight greater than $110\,\text{g}$. [3]

(d) From a random sample of 500 packets, determine the number of packets that would be expected to have a weight lying within 1.5 standard deviations of the mean. [3]

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(e) Packets are delivered to supermarkets in batches of 80. Determine the probability that at least 20 packets from a randomly selected batch have a weight less than $95\,\mathrm{g}$. [4]

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